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CLASS OF SELF-SIMILAR SOLUTIONS FOR A HIGH-TEMPERATURE AXISYMMETRIC JET

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UDC 532.526

The most simple and rigorous results in the investigation of nonisothermal jet flows of a compressible gas can be obtained by utilizing the Dorodnitsyn transformation [1]. However, this method is suitable only for plane (or nearly plane) gas flows with a linear dependence of the heat conduction and dynamic viscosity on temperature; the transition here from Dorodnitsyn to physical variables is difficult. In the case of an axisymmetric jet issuing from a point source for a domain where the temperature on the axis is considerably higher than the temperature at infinity, by using the idea of the existence of a separating layer [2], a self-similar solution can be constructed for a power-law dependence of the heat conduction and viscosity on the temperature, where it is possible to go from the initial two-parameter problem (the Prandtl number, the exponent) to a one-parameter problem.

1. We write the problem describing the emergence of a nonisothermal jet from a cylindrical orifice in the boundary-layer approximation in the dimensionless form

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\mu(T)\frac{\partial w}{\partial r}\right]=\rho\left(v\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right), \tag{1.1}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho v)+\frac{\partial}{\partial z}(\rho w)=0,\quad \rho T=1,$$
$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\lambda(T)\frac{\partial T}{\partial r}\right]=\Pr\rho\left(v\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\right);$$
$$v=\frac{\partial w}{\partial r}=\frac{\partial T}{\partial r}=0 \quad \text{for } r=0; \tag{1.2}$$

$$T=\varepsilon,\ w=0 \quad \text{for } r\to\infty, \tag{1.3}$$

where r, zR are cylindrical coordinates (r, z are internal coordinates in an asymptotic expansion in the small parameter R^{-1}), $R=\sqrt{\rho_m I_{1m}/2\pi}/\mu_m$ is a certain analog of the Reynolds number, vR^{-1}, w – r, z are velocity components; $\Pr = c_{pm}\mu_m/\lambda_m$ is the Prandtl number; and ε is the value of the temperature at infinity. The notation of the remaining quantities is standard. The scales T_m, ρ_m, c_{pm}, μ_m, λ_m (the scale quantities are marked with the subscript m), as well as the total momentum scale I_{1m} are the enthalpy flux I_{2m} defined by the formulas

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$$I_{1m} = 2\pi\rho_m V_m^2 L_m^2 \int_0^\infty \rho w^2 r dr, \quad I_{2m} = 2\pi c_{pm}\rho_m T_m V_m L_m^2 \int_0^\infty \rho w (T - \varepsilon) r dr$$

are considered given to make the quantities dimensionless. Selected as velocity V_m and L_m scales are

$$V_m = c_{pm} T_m I_{1m}/I_{2m}, \; L_m = (I_{2m}/c_{pm} T_m)/\sqrt{2\pi\rho_m I_{1m}}.$$

It was assumed in (1.1) that the specific heat is a constant. The initial condition for $z = z_0$ should be substituted for system (1.1). However, within the framework of this work only self-similar solutions will be considered, so we formulate the momentum and enthalpy flux conservation conditions to close the problem (1.1)-(1.3):

$$\int_0^\infty \rho w^2 r dr = 1, \; \int_0^\infty \rho w (T - \varepsilon) r dr = 1. \tag{1.4}$$

We consider the problem (1.1)-(1.4) for $\varepsilon \to 0$. For this case an asymptotic expansion in the small parameter ε, suitable near the boundary $r = 0$ (we later call this expansion the expansion for the hot boundary layer), can be constructed. Following [2], we assume that these expansions will not be suitable near the surface where the separating layer is localized, i.e., the domain of suitability of the expansion is in the interval $0 \leqq r < r_0(z)$, where $r = r_0(z)$ is the interface on which the temperature of the hot boundary layer equals zero in the zeroth approximation. Within the framework of this work we shall not construct the solution for the separating layer; we just note that the reasons for origination of this domain of nonuniformity and the methodology of constructing the solution in the separating layer are considered in [2]. From the physical viewpoint, a thin separating layer (compared with the boundary-layer thickness) separates the high-temperature compressible gas flow domain from the low-temperature incompressible gas flow domain with the constant temperature.

In the zeroth approximation in ε, the problem for the hot layer is described by system (1.1), boundary conditions (1.2), and the integral conditions which are now written in the form

$$\int_0^{r_0(z)} \rho w^2 r dr = 1, \; \int_0^{r_0(z)} w r dr = 1. \tag{1.5}$$

Conservation conditions (1.5) can be obtained just by assuming the existence of these integrals. The problem (1.1), (1.2), (1.5) allows self-similar solution if

$$\mu = \lambda = T^\gamma, \tag{1.6}$$

where γ is a given constant. It should be kept in mind here and below that the result is considered real and positive when raising a positive number to a noninteger power. Then, setting

$$w(r, z) = z^{\alpha_w} u(x), \; v = z^{\alpha_v} f(x), \; T = z^{\alpha_T} \theta(x), \; r = x z^\alpha, \tag{1.7}$$

where

$$\alpha_w = \alpha_T = -1/(1+\gamma), \alpha_v = -(3+2\gamma)/[2(1+\gamma)], \alpha = 1/[2(1+\gamma)], \tag{1.8}$$

we obtain from (1.1), (1.2), (1.5),

$$\frac{1}{x}(x\theta^\gamma u')' = \frac{1}{\theta}[fu' + u(\alpha_w u - \alpha x u')], \tag{1.9}$$

$$\frac{1}{x}\left(x\frac{f}{\theta}\right)' - \alpha x\left(\frac{u}{\theta}\right)' = 0, \; \frac{1}{x}(x\theta^\gamma\theta')' = \Pr\frac{1}{\theta}[f\theta' + u(\alpha_T\theta - \alpha x\theta')];$$

$$f = u' = \theta' = 0 \text{ for } x = 0; \tag{1.10}$$

$$\int_0^{x_0} (u^2/\theta) x dx = 1, \; \int_0^{x_0} u x dx = 1, \tag{1.11}$$

where the prime denotes the derivative with respect to x; x_0 is the point of separation $[\theta(x_0) = 0]$ if it exists, otherwise $x_0 \to \infty$. Let us note that $\gamma < -1$ for temperature and the longitudinal velocity on the axis grow downstream [see (1.8)]. Physically such a situation apparently cannot occur; consequently, it should be kept in mind that later only the case

$$\gamma > -1 \tag{1.12}$$

will be considered. It can generally be assumed for laminar jets that the viscosity and heat conduction grow as the temperature rises, i.e., $\gamma > 0$; however, the description of certain turbulent flows can be reduced to laminar models with $\gamma < 0$.

In connection with the fact that the problem (1.9), (1.10) is invariant to transformation

$$u \to C_1 u,\ \theta \to C_2\theta,\ x \to C_1^{-1/2}C_2^{(1+\gamma)/2}x,\ f \to G^{1/2}C_2^{(1+\gamma)/2}f, \tag{1.13}$$

we use the conditions

$$u = 1,\ \theta = 1 \ \text{for}\ x = 0 \tag{1.14}$$

as closing for the problem (1.9), (1.10). Solving the problem (1.9), (1.10), (1.14), and using the properties (1.13), the constants C_1, C_2 that satisfy the normalization conditions (1.11) can be determined. Consequently, we shall later investigate problem (1.9), (1.10), (1.14).

2. By using the transformation

$$s = \Pr(f - \alpha x u), \tag{2.1}$$

its properties and the properties of the functions θ, u following from problem (1.9), (1.10), (1.14),

$$s = \theta^\gamma\theta',\ u = \theta^{1/\Pr}, \tag{2.2}$$

we reduce this latter problem to the form

$$\frac{1}{x}(x\theta^\gamma\theta')' = \theta^{\gamma-1}\theta'^2 + \Pr\alpha_T\theta^{1/\Pr},\ \theta = 1,\ \theta' = 0 \quad \text{for} \quad r = 0. \tag{2.3}$$

Problem (2.3) has certain simple solutions. Thus, for $\gamma = 0$, Eq. (2.3) is related to the known Emden–Fowler equation [3], and the solution of (2.3) is

$$\theta = \left(1 + \frac{1-\Pr}{8}x^2\right)^{-2\Pr/(1-\Pr)}. \tag{2.4}$$

It can be obtained from the condition for the existence of integrals [conditions for the existence of self-similar solutions in the form (1.7) and (1.8)] that the solution (2.4) is acceptable for $\Pr < 3$. For $0 < \Pr \leqq 1$, the solution (2.4) is suitable in a semiinfinite interval for the variable x $(0 \leqq x < \infty)$, i.e., no separating layer originates here. For $1 < \Pr < 3$, the solution (2.4) is suitable in the interval $0 \leqq x < x_0$ and the separating layer is localized near the surface (point) of separation $x_0 = \sqrt{8/(\Pr - 1)}$.

For Pr = 1, the solution of problem (2.3) is

$$\theta = (1 + \alpha_T\gamma x^2/4)^{1/\gamma} = \left[1 - \frac{\gamma x^2}{4(1+\gamma)}\right]^{1/\gamma}. \tag{2.5}$$

This solution is applicable, if $\gamma > 0$, for $0 \leqq x < x_0 = 2\sqrt{(1+\gamma)/\gamma}$, or if $\gamma < 0$, for $0 \leqq x < \infty$. The conditions of existence of the integrals lead to a bound on γ not exceeding (1.12).

By introducing the new variables

$$\eta = x\sqrt{\Pr(-\alpha_T)|\gamma|},\ \tau = \theta^\gamma \tag{2.6}$$

we convert problem (2.3) to the form

$$\frac{1}{\eta}\frac{d}{d\eta}\left(\eta\frac{d\tau}{d\eta}\right) + \operatorname{sign}(\gamma)\tau^\beta = 0,\ \tau = 1,\ \frac{d\tau}{d\eta} = 0 \quad \text{for} \quad \eta = 0. \tag{2.7}$$

where $\beta = (1 - Pr)/(\gamma Pr)$. Therefore, the initial fifth-order two-parameter problem (γ, Pr) has successfully been converted to two [sign (γ) = ±1] one-parameter second-order problems. By solving (2.7) and using (2.6), (2.2), (2.1), self-similar functions u, θ, f of the self-similar variable x can be determined, and then, if necessary, the solution obtained can be normalized by using (1.13) and (1.11).

The problem (2.7) also has certain simple solutions. For $\beta = 0$ (Pr = 1) the solution has already been obtained [see (2.5)]. For $\beta = 1$ the solution of (2.7) is

$$\tau = J_0(\sqrt{\mathrm{sign}(\gamma)}\eta), \tag{2.8}$$

where J_0 is the Bessel function. For $\gamma > 0$ the solution (2.8) is suitable in the interval $0 \leqq \eta < k_1$, where k_1 is the least positive root of the equation $J_0(k_n) = 0$. Existence conditions for the integrals (1.11) for $\beta = 1$ and $\gamma > 0$ (Pr < 1) impose no constraints on the acceptability of solution (2.8). For $\beta = 1$ and $\gamma < 0$ (Pr > 1) solution (2.8) is suitable in the semiinfinite interval $0 \leqq \eta < \infty$, and the constraint Pr < 2 follows from the existence condition for the integrals (1.11).

The curves $\tau = \tau(\eta)$ for $\gamma > 0$ and different values of β as obtained by the Runge–Kutta numerical method are constructed in Fig. 1. Let us note that for any values of β ($-\infty < \beta < \infty$) the solutions of problem (2.7) are suitable in a bounded interval of the variable η ($0 \leqq \eta < \eta_0$). The constraints imposed on the solution of problem (2.7) can be estimated from the behavior of the function τ (or τ^{-1} for $\gamma < 0$) in the neighborhood of its zeroes, since precisely this determines the existence of the integrals (1.11). Without performing a detailed analysis, we note the final formulas describing the behavior of the function τ (or τ^{-1} for $\gamma < 0$) in the neighborhood of its zeros and the constraints following from the existence conditions for integrals (1.11). Thus, for $\gamma > 0$ and $\beta > -1$ [$(1 - \gamma)Pr < 1$], in the neighborhood of zero

$$\tau = \sqrt{\frac{2}{\beta+1}}(\eta_0 - \eta) + O((\eta_0 - \eta)^{\beta+2}),$$

from which the constraint $(1 - \gamma)Pr < 2$ follows, which is not stronger than the condition $\beta > -1$ [$(1 - \gamma)Pr < 1$]. For $\gamma > 0$ and $\beta < -1$ [$(1 - \gamma)Pr > 1$] in the neighborhood of the separation point η_0 the function τ will behave as a power law

$$\tau = \left[\frac{1-\beta}{2}\sqrt{-\frac{2}{\beta+1}}(\eta_0 - \eta)\right]^{2/(1-\beta)} + O((\eta_0 - \eta)^{1-2(\beta+1)/(1-\beta)}),$$

from which it is possible to obtain

$$(1 - \gamma)Pr < 3. \tag{2.9}$$

It follows from condition (2.9) and the behavior of the function $\beta > -1$ that the solutions of problem (2.7) are suitable for any values of Pr if $\gamma \geqq 1$. For $\gamma > 0$ and $\beta = -1$ (Pr > 1) and asymptotic estimate yields in the neighborhood of the separation point

$$\eta_0 - \eta = \frac{\tau}{\sqrt{-2\ln\tau}}(1 + O(\ln^{-1}\tau)),$$

from which it follows that the integrals (1.11) exist for any values of γ (or Pr).

Solutions of the problem (2.7) are presented in Fig. 2 for $\gamma < 0$ and different values of the parameter β. For $\beta > 1$ [$(1 + \gamma)Pr > 1$] and $\gamma < 0$ the solution of problem (2.7) for is suitable in a bounded domain of the variable η ($0 \leqq \eta < \eta_0$) and will behave in the neighborhood of a zero of the function τ^{-1} as

$$\tau = \left[\frac{\beta-1}{2}\sqrt{\frac{2}{\beta+1}}(\eta_0 - \eta)\right]^{2/(1-\beta)} + O((\eta_0 - \eta)^{1-2(\beta+1)/(1-\beta)}),$$

while the existence conditions for the integrals (1.11) will result in the constraint (2.9). For $\gamma < 0$ and $-1 < \beta < 1$ the solution for τ is suitable in the semiinfinite interval $0 \leqq \eta < \infty$, while for $\eta \to \infty$ the function τ behaves as

$$\tau = \left(\frac{1-\beta}{2}\sqrt{\frac{2}{\beta+1}}\eta\right)^{2/(1-\beta)} + O(\eta^{-1}) + O(\eta^{-(2+2\beta)/(1-\beta)}).$$

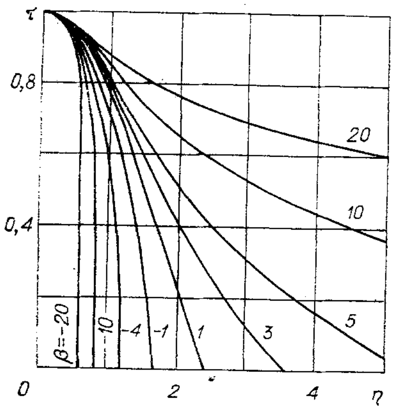

Fig. 1

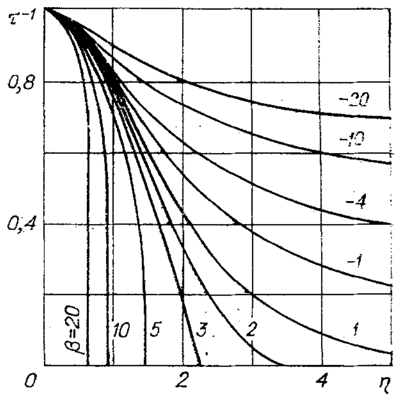

Fig. 2

Then for $\gamma < 0$ and $-1 < \beta < 1$ it is possible to obtain $\Pr < -1/\gamma$ from the existence conditions for the integrals (1.11). For $\beta > 1$ and $\eta \to \infty$ the function $\tau \sim \ln \eta$ and the integrals (1.11) do not exist. Therefore, the self-similar representation (1.7), (1.8) is not suitable for $\gamma < 0$ and $\beta < -1$.

Let us note that the existence conditions for the integrals (1.11) allow infinite derivatives in η for the longitudinal velocity or temperature in the neighborhood of the separation point. For instance, for $\Pr = 2$ and $\gamma = 3/2$ near the point $\eta = \eta_0$, $u \sim (\eta_0 - \eta)^{1/3}$, $\theta \sim (\eta_0 - \eta)^{2/3}$.

3. Let us investigate the solution of problem (2.7) in the case of β, which is large in absolute value. The terminology and ideas (particularly the most powerful perturbation theory method, in our opinion, merger at an intermediate limit) of merging asymptotic expansions will be used in constructing the solution [4]. It is expedient to introduce the new function

$$g = \tau^{\beta-1} \tag{3.1}$$

to convert the problem (2.7) to the form

$$\frac{1}{\eta}\frac{d}{d\eta}\left(\eta\frac{dg}{d\eta}\right) - (1-\varepsilon_1)\frac{1}{g}\left(\frac{dg}{d\eta}\right)^2 + \frac{\operatorname{sign}(\gamma)}{\varepsilon_1}g^2 = 0, \tag{3.2}$$

$$g = 1,\ \frac{dg}{d\eta} = 0 \quad \text{for} \quad \eta = 0,$$

where $\varepsilon_1 = 1/(\beta - 1)$.

We consider the problem (3.2) first in the case $0 < \varepsilon_1 \ll 1$. We formulate the internal limit process in the form

$$\varepsilon_1 \to 0\ (\beta \to \infty),\ \zeta = \eta/\sqrt{\varepsilon_1} \text{ is fixed.} \tag{3.3}$$

We then construct the internal expansion of the solution of problem (3.2) as

$$g(\eta, \varepsilon_1) = g_0(\zeta) + \nu_1(\varepsilon_1)g_1(\zeta) + \ldots \nu_n(\varepsilon_1)g_n(\zeta) + \ldots (n = 1, 2, \ldots), \tag{3.4}$$

where $\nu_n(\varepsilon_1)$ is a certain asymptotic sequence. Substituting (3.4) into problem (3.2), and keeping in mind the definition of the internal limit (3.3), we obtain

$$\frac{1}{\zeta}\frac{d}{d\zeta}\left(\zeta\frac{dg_0}{d\zeta}\right) - \frac{1}{g_0}\left(\frac{dg_0}{d\zeta}\right)^2 + g_0^2 = 0,\ g_0 = 1,\ \frac{dg_0}{d\zeta} = 0 \quad \text{for} \quad \zeta = 0. \tag{3.5}$$

which is related to the Emden–Fowler equation [3]. The solution of (3.5) is

$$g_0 = (1 + \zeta^2/8)^{-2}. \tag{3.6}$$

The solution of problem (3.2) in form (3.6) is suitable with confidence only for arbitrarily small η since it is evident from the above that for $\beta > 0$ and $\gamma > 0$ a separation point exists (a zero of the function τ). Therefore, the suitability of the expansion (3.5) in the neighborhood of an η not arbitrarily small [i.e., in the case when $\eta \to 0$ not as rapidly as at the internal limit (3.3)] is already questionable. Consequently, we formulate the external limit process

$$\tilde{\zeta} = \eta/\delta(\varepsilon_1) \quad \text{is fixed}, \quad \delta(\varepsilon_1)/\sqrt{\varepsilon_1} \to \infty \quad \text{for} \quad \varepsilon_1 \to 0. \tag{3.7}$$

Here $\delta(\varepsilon_1)$ characterizes the scale of the variable η at the external limit, but the function $\delta(\varepsilon_1)$ is still not defined.

It is seen from problem (2.7) that as $\beta \to \infty$ and $0 < \tau < 1$ (the latter follows from temperature positivity and monotoneity considerations), its solution will behave as

$$\tau = A + B \ln \eta + \mathrm{TST}\,(\beta^{-1})$$

to an error of transcendentally small terms for at least large or not too small values of η, where A, B are constants independent of η. Consequently, keeping in mind (3.1) and the definition of the parameter ε_1 it can be assumed that the expansion for the function g at the external limit (3.7) has the form

$$g(\eta, \varepsilon_1) = [A_0\tilde{\nu}_0(\varepsilon_1) + \ldots A_n\tilde{\nu}_n(\varepsilon_1) + \ldots]\{1 + [B_0\tilde{\mu}_0(\varepsilon_1) + \ldots B_n\tilde{\mu}_n(\varepsilon_1) + \ldots]\ln \eta\}^{1/\varepsilon_1}, \tag{3.8}$$

where A_n, B_n are constants independent of ε_1; $\tilde{\nu}_n(\varepsilon_1)$, $\tilde{\mu}_n(\varepsilon_1)$ are asymptotic sequences. Without limiting the generality of the external limit process and of the expansion (3.7) itself, it was assumed in writing (3.8) that

$$\delta(\varepsilon_1) = 1. \tag{3.9}$$

We merge the expansions (3.4) and (3.8) at a limit intermediate between the limits (3.3) and (3.7). We formulate the intermediate limit process in the form

$$\varkappa \to 0, \ \varkappa/\sqrt{\varepsilon_1} \to \infty, \ \eta_\varkappa = \eta/\varkappa \quad \text{is fixed for} \quad \varepsilon_1 \to 0. \tag{3.10}$$

Then, by merging (3.4) and (3.8) at the limit (3.10), we have

$$\left(1 + \frac{\eta_\varkappa^2\varkappa^2}{8\varepsilon_1}\right)^{-2} + \nu_1(\varepsilon_1)\, g_1(\eta_\varkappa\varkappa/\sqrt{\varepsilon_1}) + \ldots = [A_0\tilde{\nu}_0(\varepsilon_1) + \ldots]\{1 + [B_0\tilde{\mu}_0(\varepsilon_1) + \ldots]\ln \eta_\varkappa\varkappa\}^{1/\varepsilon_1},$$

from which we successively obtain

$$\tilde{\mu}_0 = \varepsilon_1, \ \tilde{\nu}_0 = \varepsilon_1^2, \ B_0 = -4, \ A_0 = 64$$

and in a zeroth approximation in ε_1 at the external limit (3.7), (3.9)

$$g = 64\varepsilon_1^2(1 - 4\varepsilon_1 \ln \eta)^{1/\varepsilon_1},$$

i.e., the separating layer occurs near the point $\eta = \eta_0 = \exp(1/4\varepsilon_1)$ here.

The solution of problem (3.2) is constructed in an analogous manner in the case sign $(\gamma) = -1$ as $\beta \to -\infty$. A uniform approximation can be obtained by combining solutions suitable at the internal and external limits and subtracting their common part. Then in the case $\gamma\beta > 0$ for $\beta \to \pm\infty$ $(\varepsilon_1 \to \pm 0)$

$$g = \left(1 + \frac{\eta^2}{8|\varepsilon_1|}\right)^{-2} + 64\varepsilon_1^2(1 - 4\varepsilon_1 \ln \eta)^{1/\varepsilon_1} - \frac{64\varepsilon_1^2}{\eta^2}.$$

will be a uniformly suitable approximation in ε_1 and η for the problem (3.2).

For $\gamma\beta < 0$ and $\beta \to \pm\infty(\varepsilon_1 \to \pm 0)$ an expansion of the form (3.4) will be a limit process in the form

$$\varepsilon_1 \to \pm 0(\beta \to \pm\infty), \ \zeta = \eta/\sqrt{|\varepsilon_1|} \ \text{is fixed.}$$

will be uniformly suitable for problem (3.2). In this case solution of (3.2) in a zeroth approximation in ε_1 has the form

$$g(\eta, \varepsilon_1) = \left(1 - \frac{\eta^2}{8|\varepsilon_1|}\right)^{-2} + O(\varepsilon_1).$$

The separating layer here evidently occurs for $\eta = \eta_0 = \sqrt{8|\varepsilon_1|}$.

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COATING OF A NON-NEWTONIAN FLUID ONTO A MOVING SURFACE

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The application of a coating of non-Newtonian fluids to a moving surface was considered in [1-4]. These studies are based on the approach proposed in [5, 6], which is restricted by the requirement that the thickness h_0 of the coated film be small as compared with the capillary constant $(\sigma/\rho g)^{1/2}$ (ρ is the density, g the free-fall acceleration, and σ the surface tension). Experimental studies of fluid coating [1, 7] have revealed substantial differences between the theoretical and experimental data. Thus at present we lack a theory that satisfactorily describes the coating of non-Newtonian fluids. In this article such a theory is developed for fluids with nonlinear viscosity.

1. Let us consider the process of coating a liquid onto a vertical surface moving at a constant speed (Fig. 1). Because of the action of gravity, the withdrawn plate entrains only part of the liquid it sets in motion. Accordingly, on the free surface there is a stagnation line (perpendicular to the plane of the drawing) where the velocity is equal to zero and the flow direction branches [8]. The streamlines passing through the stagnation line separate the wall zone of liquid entrained by the wall from that remaining in the bath.

We take the stagnation line as the origin and direct the x axis vertically upward in the direction of motion of the surface, and the y axis at right angles to the latter. The flow region bounded below by a plane perpendicular to the wall and passing through the stagnation line and tending upward to the constant thickness h_0 we will call the dynamic meniscus zone. Clearly, the length L of the dynamic meniscus zone considerably exceeds its width h_0; this naturally gives rise to the small parameter $\varepsilon = h_0/L \ll 1$. Consequently, the variation of the characteristics along the x axis is much weaker than in the transverse y direction, i.e., the derivatives with respect to y are much greater than those with respect to x. Making the appropriate estimates [9] in the equations of motion and the boundary conditions, in the region of the dynamic meniscus we obtain

$$\partial\tau/\partial y + \rho g - \partial p/\partial x = 0,\ \partial p/\partial y = 0; \tag{1.1}$$

$$u = U \text{ when } y = 0,\ \tau = 0 \text{ when } y = h; \tag{1.2}$$

$$p - p_0 = -\sigma d^2h/dx^2 \text{ when } y = h. \tag{1.3}$$

We represent the continuity equation in integral form: $Q = \int_0^h u\,dy =$ const. Here τ is the shear stress due to friction; h is the coordinate of the free surface of the liquid; p is the pressure in the liquid; u is the x component of the velocity vector; p_0 = const is the pressure in the gas; and Q is the rate of flow of the liquid in the film.

Integrating (1.1) with respect to y and using (1.2) and (1.3), we find

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